MULTIDIMENSIONAL SOIL EROSION/DEPOSITION MODELING

PART III: Process based erosion simulation

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Prepared for:

U.S.Army Construction Engineering Research Laboratories

Contract No.: DACA88-96-Q-0314

Urbana, December 1996

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1. Introduction

Recent advances in Geographic Information Systems (GIS) technology, especially support for modeling with multi-variate fields [Mitasova et al., 1995a; Mitas et al., 1996], along with the exponential growth in computational power, stimulate the shift from empirical, lumped models to physically-based, distributed ones. A typical example of this trend is the current development in hydrologic and erosion modeling [Maidment, 1996, Moore et al., 1993]. Integration of distributed hydrologic models within GIS [Vieux et al., 1996; Saghafian, 1996] improved capabilities to estimate overland flow in complex landscapes. The development of analogous distributed simulation tools for erosion, sediment transport and deposition by overland flow has been slower, mostly because of the enormous complexity of the processes involved.

While important progress towards physically based erosion simulations was made by the introduction of a new generation erosion prediction tool within the Water Erosion Prediction Project (WEPP) [Foster et al., 1995], and by linking the existing simulation programs with GIS [Engel, 1995; Rewerts and Engel, 1991; Srinivasan and Engel, 1991], several problems persist. Most models, including the distributed ones, are based on a 1 D sediment routing over planar hillslope segments [Hairsine and Rose, 1992; Govindaraju and Kavvas, 1991; Foster et al., 1995; Flacke et al., 1990] or through homogeneous (lumped) subwatersheds [Arnold et al., 1993; Srinivasan and Arnold, 1994]. Due to these simplifications, most of the currently used models can only partially explain the impact of a complex spatial variability in terrain and land cover at a landscape scale, where even small variations can have a dramatic impact on location and rates of erosion and deposition. Unit stream power based methods and related approaches [Moore and Burch, 1986; Moore and Wilson, 1992; Mitasova et al., 1996a; Desmet and Govers, 1995; Hofierka and Suri, 1996] partially incorporated the influence of complex terrain. While this simple approach provides useful estimates of topographic effects on erosion/deposition, it does not simulate the impact of a wide range of soil and cover properties, typical for anthropogenic landscapes

[Desmet and Govers, 1995]. Several distributed landscape evolution models also describe erosion processes [e.g., Kirkby, 1986; Willgoose et al., 1991; Kramer and Marder, 1992; Howard, 1994], however, their application to land management is rather limited because these models are designed for time scales of thousands of years and focus on the evolution of topography and stream networks, rather than on predictions of erosion risk locations and erosion prevention.

To achieve the realism and effectiveness necessary for land management applications, the erosion simulation tools should: i) minimize reliance on empirical concepts and factors and employ fundamental physically based equations and parameters with well defined physical interpretation, ii) use robust numerical methods capable of supporting simulations at high spatial and temporal resolutions, iii) fully incorporate the influence of spatial and temporal variability in rainfall, terrain, cover, and soils.

In this project we have developed a methodology for erosion simulations aimed at a gradual fulfillment of the above defined requirements.

2. Process based erosion simulation (Mitas et al. 1996)

The methodological framework for the simulation of erosion/deposition processes is based upon the description of water flow and sediment transport processes by first principles equations, a concept outlined previously, most often for a one dimensional case, for example by Foster and Meyer, [1972] or Bennet, [1994]. Within our approach, inputs and outputs of the simulations are represented by multi-variate functions (scalar or vector fields) as genuine distributed objects. Advanced GIS technology is used to support the processing, analysis and visualization of these fields [Mitasova et al., 1995a; Mitas et al., 1996; Mitas et al., 1997; Brown et al., 1995] The implementation of the process based erosion simulation developed by Mitas et al. (1996), Mitas et al. (1997) and Mitas and Mitasova (subm) in form of computational tools is called SIMWE: **SIM**ulation of **W**ater **E**rosion.

2.1 Overland water flow

A 2D shallow water flow is described by the bivariate form of Saint Venant equations [e.g., Julien et al., 1995]. The continuity relation is given by

$$\frac{\partial h(\mathbf{r},t)}{\partial t} = i_e(\mathbf{r},t) - \nabla \cdot \mathbf{q}(\mathbf{r},t) \quad (1)$$

while the momentum conservation in the diffusive wave approximation has the form

$$\mathbf{s_f}(\mathbf{r},t) = \mathbf{s}(\mathbf{r}) - \nabla h(\mathbf{r},t) \quad (2)$$

where $\mathbf{r} = (x,y)$ $[m]$ is the position, t $[s]$ is the time, $h(\mathbf{r},t)$ $[m]$ is the depth of overland flow, $i_e(\mathbf{r},t)$ $[m/s]$ is the rainfall excess, $\mathbf{q}(\mathbf{r},t)$ $[m^2/s]$ is the unit flow discharge (water flow per unit width), $\mathbf{s}(\mathbf{r}) = -\nabla z(\mathbf{r})$ is the negative elevation gradient, $z(\mathbf{r})$ $[m]$ is the elevation, and $\mathbf{s_f}(\mathbf{r},t)$ is the negative gradient of overland flow surface (friction slope). For a shallow water overland flow, with the hydraulic radius approximated by the normal flow depth $h(\mathbf{r},t)$ [Moore and Foster, 1990], the unit discharge is given by

$$\mathbf{q}(\mathbf{r},t) = \mathbf{v}(\mathbf{r},t)h(\mathbf{r},t) \quad (3)$$

where $\mathbf{v}(\mathbf{r},t)$ $[m/s]$ is the flow velocity. The system of equations (1-3) is closed using the Manning's relation between $h(\mathbf{r},t)$ and $\mathbf{v}(\mathbf{r},t)$

$$\mathbf{v}(\mathbf{r},t) = \frac{C}{n(\mathbf{r})} h(\mathbf{r},t)^{2/3} |\mathbf{s_f}(\mathbf{r})|^{1/2} \mathbf{s_{f0}}(\mathbf{r}) \quad (4)$$

where $n(\mathbf{r})$ is the dimensionless Manning's coefficient, $C = 1$ is the corresponding dimension constant $[m^{1/3}/s]$ [Dingman, 1984], and $\mathbf{s_{f0}}(\mathbf{r}) = \mathbf{s_f}(\mathbf{r})/|\mathbf{s_f}(\mathbf{r})|$ is the unit vector in the friction slope direction. In this project, we assume that the solution of continuity and momentum equations for a steady state $\partial h(\mathbf{r},t)/\partial t = 0$, provides an adequate estimate of overland flow for the land management applications [Flanagan and Nearing, 1995]. In addition, we assume that the flow is close to the kinematic wave approximation for which $\mathbf{s_f}(\mathbf{r},t) \approx \mathbf{s}(\mathbf{r})$ and after using (3), the equation (1) is given by

$$\nabla \cdot [h(\mathbf{r})\mathbf{v}(\mathbf{r})] = i_e(\mathbf{r}) \quad (5)$$

In order to incorporate, at least in an approximate way, the diffusive wave effects we include a diffusion-like term $\propto \nabla^2[h^{5/3}(\mathbf{r})]$ into (5) which is then given by

$$-\frac{\varepsilon}{2}\nabla^2[h^{5/3}(\mathbf{r})] + \nabla \cdot [h(\mathbf{r})\mathbf{v}(\mathbf{r})] = i_e(\mathbf{r}) \tag{6}$$

where ε is a diffusion coefficient. Such an incorporation of diffusion in the water flow simulation is not new and a similar term has been obtained in derivations of diffusion-advection equations for overland flow, e.g., by Dingman, [1984] and Lettenmeier and Wood, [1992]. In our reformulation, we simplify the diffusion coefficient to a constant and we use a modified diffusion term, which depends on $h^{5/3}(\mathbf{r})$ instead of $h(\mathbf{r})$. The equation (6) has the advantage of being *linear* in the function $h^{5/3}(\mathbf{r})$ (see equation (4)) which enables us to solve it by means of the Green's function method using stochastic (Monte Carlo) techniques as described later. The diffusion constant which we have used is rather small (approximately one order of magnitude smaller than the reciprocal Manning's coefficient) and therefore the resulting flow is close to the kinematic regime. However, the diffusion term improves the kinematic solution, by overcoming small shallow pits common in digital elevation models (DEM) and by smoothing out the impact of slope discontinuities or abrupt changes in Manning's coefficient (e.g., due to a road, or other anthropogenic changes in elevations or cover).

2.2 Sediment flow

The basic relationship describing the sediment transport by overland flow is the continuity of sediment mass, which relates the change in sediment storage over time, and the change in sediment flow rate along the hillslope to effective sources and sinks [e.g., Haan et al., 1994; Govindaraju and Kavvas, 1991; Foster and Meyer, 1972; Bennet, 1974]. The bivariate form of the continuity of sediment mass equation is [e.g., Hong and Mostaghimi, 1995]:

$$\frac{\partial[\rho_s c(\mathbf{r},t)h(\mathbf{r},t)]}{\partial t} + \nabla \cdot \mathbf{q_s}(\mathbf{r},t) = \text{sources} - \text{sinks} = D(\mathbf{r},t) \tag{7}$$

where $\mathbf{q_s}(\mathbf{r},t)$ $[kg/(ms)]$ is the sediment flow rate per unit width, $c(\mathbf{r},t)$ [particle$/m^3$] is sediment concentration, ρ_s $[kg/$particle$]$ is mass per sediment particle, $\rho_s c(\mathbf{r},t)$ $[kg/m^3]$ is sediment mass density, and $D(\mathbf{r},t)$ $[kg/(m^2 s)]$ is the net erosion or deposition rate. The sediment flow rate $\mathbf{q_s}(\mathbf{r},t)$ is a function of water flow and sediment concentration:

$$\mathbf{q_s}(\mathbf{r},t) = \rho_s c(\mathbf{r},t)\mathbf{q}(\mathbf{r},t) \tag{8}$$

Again, we assume a steady state form of the continuity equation:

$$\frac{\partial[\rho_s c(\mathbf{r},t)h(\mathbf{r},t)]}{\partial t} = 0 \quad \longrightarrow \quad \nabla \cdot \mathbf{q_s}(\mathbf{r}) = D(\mathbf{r}). \tag{9}$$

The sources/sinks term is derived from the assumption that the detachment and deposition rates are proportional to the difference between the sediment transport capacity and the actual sediment flow rate [Foster and Meyer, 1972]:

$$D(\mathbf{r}) = \sigma(\mathbf{r})\left[T(\mathbf{r}) - |\mathbf{q_s}(\mathbf{r})|\right] \tag{10}$$

where $T(\mathbf{r})$ $[kg/(ms)]$ is the sediment transport capacity, and $\sigma(\mathbf{r})$ $[m^{-1}]$ is the first order reaction term dependent on soil and cover properties. The $\sigma(\mathbf{r})$ is obtained from the following relationship [Foster and Meyer, 1972]:

$$D(\mathbf{r})/D_c(\mathbf{r}) + |\mathbf{q_s}(\mathbf{r})|/T(\mathbf{r}) = 1 \tag{11}$$

which states that the ratio of erosion rate to the detachment capacity $D_c(\mathbf{r})$ $[kg/(m^2 s)]$ plus the ratio of the sediment flow to the sediment transport capacity is a conserved quantity (unity). The sediment transport capacity $T(\mathbf{r})$ and detachment capacity $D_c(\mathbf{r})$ represent maximum potential sediment flow rate and maximum potential detachment rate, respectively, and are functions of a shear stress [Foster and Meyer, 1972] The parameters and adjustment factors for the estimation of $D_c(\mathbf{r}), T(\mathbf{r}), \sigma(\mathbf{r})$ are functions of soil and cover properties, and their values for a wide range of soils, cover, agricultural and erosion prevention practices are being developed within the WEPP model [Flanagan and Nearing, 1995].

To solve the bivariate continuity equation (9), we at first define a function $\varrho(\mathbf{r})$ $[kg/m^2]$ as:

$$\varrho(\mathbf{r}) = \rho_s c(\mathbf{r}) h(\mathbf{r}) \tag{14}$$

Introducing a small diffusion term $\propto \nabla^2 \varrho(\mathbf{r})$, which represents local dispersion processes of the suspended flow [Bennet, 1974], for example, the impact of small, local slope changes below the DEM resolution, we rewrite the continuity equation as

$$-\frac{\omega}{2}\nabla^2 \varrho(\mathbf{r}) + \nabla \cdot [\varrho(\mathbf{r})\mathbf{v}(\mathbf{r})] + \varrho(\mathbf{r})\sigma(\mathbf{r})|\mathbf{v}(\mathbf{r})| = \sigma(\mathbf{r})T(\mathbf{r}) \tag{15}$$

where $\omega[m^2/s]$ is the diffusion constant. On the left hand side of the equation (15) the first term describes local diffusion, the second term is a drift driven by the water flow while the third term represents a velocity dependent 'potential' acting on $\varrho(\mathbf{r})$. The size of the diffusion constant used was again about one order of magnitude smaller than the reciprocal Manning's constant so that the impact of the diffusion term was relatively small.

2.3 Green's function stochastic method of solution

The equations (6) and (15) have a similar form in which a linear differential operator $\mathcal{O}$ acts on a nonnegative function $\gamma(\mathbf{r})$ (either $h(\mathbf{r})$ or $\varrho(\mathbf{r})$), while on the left hand side, there is a source term $\mathcal{S}(\mathbf{r})$

$$\mathcal{O}\gamma(\mathbf{r}) = \mathcal{S}(\mathbf{r}) \tag{16}$$

Thus the solution can be symbolically written as

$$\gamma(\mathbf{r}) = \mathcal{O}^{-1}\mathcal{S}(\mathbf{r}) \tag{17}$$

or explicitly, using the Green's function

$$\gamma(\mathbf{r}) = \int_0^\infty \int G(\mathbf{r}, \mathbf{r}', p)\mathcal{S}(\mathbf{r}')d\mathbf{r}'dp \tag{18}$$

The Green's function $G(\mathbf{r}, \mathbf{r}', p)$ is given by the following equation and an initial condition

$$\frac{\partial G(\mathbf{r}, \mathbf{r}', p)}{\partial p} = -\mathcal{O}G(\mathbf{r}, \mathbf{r}', p)\,; \quad G(\mathbf{r}, \mathbf{r}', 0) = \delta(\mathbf{r} - \mathbf{r}') \tag{19}$$

where δ is the Dirac function. In addition, we assume that the spatial region is a delineated watershed with zero boundary condition which is fulfilled by $G(\mathbf{r}, \mathbf{r}', p)$. The corresponding equations can be solved, e.g., by projection methods [Rouhi and Wright, 1995]. Another equivalent alternative is to interpret equations (6),(15) and (16) as describing stochastic processes with diffusion and drift components (Fokker-Planck equations) and carry out the actual simulation of the underlying process utilizing stochastic methods [Gardiner, 1985]. This is very similar to Monte Carlo methods in computational fluid dynamics or to quantum Monte Carlo approaches for solving the Schrödinger equation [Schmidt and Ceperley, 1992, Hammond et al., 1994; Mitas, 1996]. The solution by stochastic approach is illustrated by an animation in Mitas et al., [1997], (Figure 1).

The Monte Carlo technique has several unique advantages which are becoming even more important due to new developments in computer technology. Perhaps one of the most significant Monte Carlo properties is robustness which enables us to solve the equations for complex cases, such as discontinuities in the coefficients of differential operators (in our case, abrupt slope or cover changes, etc). Also, rough solutions can be estimated rather quickly, which allows us to carry out preliminary quantitative studies or to rapidly extract qualitative trends by parameter scans. In addition, the stochastic methods are tailored to the new generation of computers as they provide scalability from a single workstation to large parallel machines due to the independence of sampling points. Therefore, the methods are useful both for everyday exploratory work using a desktop computer and for large, cutting-edge applications using high performance computing.

2.4 Impact of uniform soil and cover properties

We have tested the presented approach using the data from experimental farm in Germany. The area and the field measurements are described by Auerswald et al., [1996] and by Warren et al., [1996], the data processing, analysis and visualization was described in our previous reports [Mitasova et al. 1995b, 1996b, PART I, PART II]. Analysis of the impact of terrain on spatial distribution of erosion and deposition is described by Mitas and Mitasova [submitted].

In this report we focus on soil and cover impact as important factors related to land management and land rehabilitation. Soil and cover properties influence erosion, sediment transport and deposition by their impact on flow velocity, soil detachability and transportability. These effects are in our model represented by the Manning's roughness coefficient n, effective detachment capacity coefficient (adjusted erodibility) K_d, and effective sediment transport capacity coefficient K_t. The location and extent of deposition is controlled by the first-order reaction coefficient σ in (10), related to the soil detachability and transportability. There are two limiting cases of erosion and sediment transport [Foster and Meyer, 1972; Hairsine and Rose, 1992]: i) **detachment limited** and ii) **sediment transport capacity limited**.

The first case is represented by $\sigma \rightarrow 0$, which, after substituting into (9), (10) results in the net erosion $D(\mathbf{r})$ equal to the detachment capacity given by (13). Therefore, for conditions when $\sigma << 1$ (e.g., soils and cover with $K_t >> K_d$), there is a prevailing detachment limited erosion, with almost all the detached sediment transported to the stream and with deposition restricted to small concave areas and channels (Table 1, Figure 2). This case for a 1D hillslope is modeled by the Universal Soil Loss Equation (USLE).

For the second limiting case $\sigma \rightarrow \infty$, which, after substituting into (10), leads to the $|\mathbf{q}_s(\mathbf{r})| \approx T(\mathbf{r})$, the net erosion/deposition is modeled as a divergence of the sediment transport capacity. In fact, this limit is effectively reached for $\sigma \geq 1$ (e.g., soils and cover with $K_t \leq K_d$) when the model predicts large extent of areas with deposition (Table 1, Figure 3). Such a behavior is close to the observed distribution of colluvial deposits (Figure 6), suggesting the prevailing influence of the transport capacity limited case on a long term pattern of deposition. This case is modeled by the Unit Stream Power based erosion/deposition model (USPED) described in the PART II report (Mitasova et al. 1996b).

The results of simulations for the σ values increasing from 0.01 (fine soils) to 10.0 (sandy soils), assuming a rough cover (grass) with $n = 0.1$ (Table 1), demonstrate that with the changing σ, the spatial distribution of erosion and deposition over the landscape changes dramatically. The erosion and sediment transport shows a transition between the two limiting cases (detachment and transport capacity limited). With the increasing σ, the deposition areas expand while the sediment flow rate in the streams decreases until the pattern typical for a transport capacity limiting case is reached (Table 1).

Table 1. *Input parameters and summary results of simulations for different uniform cover and soil properties represented by different values of Mannings coefficient n, detachment capacity coefficient K_d $[s/m]$, transport capacity coefficient K_t $[s]$, and first order reaction coefficient σ $[m^{-1}]$. The influence of these parameters is illustrated by resulting maximum sediment transport capacities T_{max} $[kg/(ms)]$ and maximum sediment flow rates $q_{s,max}$ $[kg/(ms)]$, by the total erosion and deposition rates over the entire area $\sum D^+$ and $\sum D^-$ $[kg/(m^2s)]$ respectively, and by the changes in the % area P_d, P_e experiencing net deposition and net erosion respectively.*

Soil and cover cond.	clay/grass	silt/grass	ssilt/grass	sand/grass	ssilt/bare	ssilt/gr*
n	0.1	0.1	0.1	0.1	0.01	0.1
K_d	0.0001	0.0003	0.001	0.003	0.03	0.001
K_t	0.01	0.003	0.001	0.0003	0.03	0.001
σ	0.01	0.1	1.0	10.	1.0	1.0
T_{max}	26.0	8.0	2.7	0.8	4.1	0.2
$q_{s,max}$	21.0	6.8	2.7	0.8	3.8	0.2
$\sum D^- / \sum D^+$	-888/951	-241/276	-87/95	-26/29	-443/456	-51/56
$P_d/P_e[\%]$	7/93	15/85	24/76	28/72	14/86	37/63
$\sum D$	63.	25.	8.	3.	13.	4.

The location and magnitude of deposition is also influenced by the overland flow velocity, which depends on a surface roughness represented by the Manning's n. For the same value of $\sigma = 1.0$, the extent of deposition for smooth surfaces (e.g. bare soil with $n = 0.01$) is smaller than for rough surfaces (e.g. grass with $n = 0.1$), assuming also an increase in detachability and transportability for bare soil as compared to grass (Figures 3, 4, Table 1).

In an attempt to reach the extent of deposition close to the 40% area, indicated by the observed data (Table 1, Figure 6), we have tried to use a modified equation for the transport capacity $T(\mathbf{r}) = K_t(\mathbf{r})\rho_w\, g[h(\mathbf{r})]^{0.6}[\sin\beta(\mathbf{r})]^{1.3}$ which has lower values of exponents than those suggested by WEPP [Foster et al., 1995] (Table 1, Figure 5). Arguably, the large percentage of deposition areas observed can be caused by time averaging effects in which rainfall events of various intensities cause accumulation of deposits over a larger area than one can estimate from a single event. Seasonal changes in land cover could have a similar impact, too. Therefore the small exponent might reflect the time averaging effect rather than a description of a single event. Clearly this deserves a systematic study supported by appropriate experiments.

2.5 Impact of spatially variable cover

While terrain plays an important role in spatial distribution of erosion/deposition, and in general we find deposition in concave areas and erosion in convex areas, this pattern can change significantly due to the spatial variability of land cover and soil properties. Borders between different land covers (e.g., bare soil and dense grass) cause abrupt changes in flow velocities, as well as in transport and detachment capacities, creating effects important for erosion prevention. We illustrate these effects for the original, conventional land use design in a subset of the study area, which combines arable land and meadows (Figure 7). We performed a simulation with the parameters set for dense grass in the meadow area $(n = 0.1, K_t = K_d = 0.0003)$ and bare soil in the arable area $(n = 0.01, K_t = K_d = 0.03)$, and compared the results with the observed spatial patterns of erosion/deposition.

The results of the simulation show several important phenomena. The highest rates of both the net erosion and the net deposition are predicted in valleys with high concentrated sediment flow (Figures 8, 9), a phenomenon typical for thalweg erosion. Field measurements confirm that this area has the thickest layers of colluvial deposits with large linear erosion features observed after an unusually strong storm (Figure 7), [Auerswald et al., 1996]. About a magnitude lower, but still relatively high erosion rates were predicted on bare upper parts of hillslopes which had the highest loss of radio-tracers and the lowest yields (Mitasova et al. 1996b, PART II), and where initiation of dense rilling occurred. Another area of increased erosion is predicted for narrow stripes below the grass areas, where water accelerates after depositing the sediment and leaving the grass. This leads to an increase in the difference $T(\mathbf{r}) - |\mathbf{q}_s(\mathbf{r})|$, creating the conditions for higher net erosion.

Deposition is predicted at the lower, concave parts of hillslopes which is in agreement with the spatial distributions of colluvial deposits (Figure 6), and radio-tracers [Mitasova et al. 1996b, PART II]. The sediment flow rate sharply decreases and eroded material deposits at the upper edges of meadows, explaining the cut-off rills with unusual shapes, which follow the borderline between the grass and bare soil areas (Figures 7, 8, 9). The influence of grass cover on flow velocity, transport and detachment capacities, also explains the observed location of deposited material on the convex, upper part of the hillslope with a meadow, however, to properly simulate the impact on the depth of colluvial deposits, long term data on rainfall and especially land cover would be needed.

Because the rill initiation process is not included within the presented model, there are a few small concave areas with observed dense rills where we have predicted high sediment flow rates with net deposition, however, the actual rilling process might have resulted in a net erosion. Distribution of rills indicates a detachment limited case of erosion during the large storm leading to an erosion pattern illustrated by the Figure 2, as opposed to the sediment transport limited erosion modeled in Figure 3.

3. Processing of soil parameters for erosion modeling

Numerous soil and cover properties influence the surface runoff and erosion/deposition process. The water flow is significantly influenced by infiltration and surface roughness. In this report we consider the case with spatially uniform infiltration or with fully saturated soil. Estimation of water depth under conditions of spatially variable infiltration can be performed by other hydrologic models, such as r.hydro.CASC2d (Mitasova et al. 1995b, PART I), which also includes the description of necessary soil parameters (Mitasova et al. 1995b, PART I, Appendix). Surface roughness is in our implementation represented by Mannings coefficient, for which values for wide range of surfaces are available (e.g. Haan et al. 1994). An analysis of the suitability of the published values of this coefficient to surface roughness generated by military use needs to be tested to ensure the validity of the simulation results for specific conditions at military installations.

Soil and cover parameters for estimation of soil detachment and sediment transport are more complex and less understood, however, there is an ongoing experimental research related to the development of WEPP, which has a potential for significant improvement in the development of physically based parameters.

3.1 Parameters representing soil and cover

Sediment flow rate and consequently net erosion/deposition is influenced by detachment capacity $D_c(\mathbf{r})$ and transport capacity $T(\mathbf{r})$ which are functions of a shear stress [Foster and Meyer, 1972]. For erosion by overland flow these capacities are expressed by simplified equations:

$$T(\mathbf{r}) = K_t(\mathbf{r})\big[\tau(\mathbf{r})\big]^p = K_t(\mathbf{r})\big[\rho_w\, g h(\mathbf{r}) \sin\beta(\mathbf{r})\big]^p \qquad (12)$$

$$D_c(\mathbf{r}) = K_d(\mathbf{r})\big[\tau(\mathbf{r}) - \tau_{cr}(\mathbf{r})\big]^q = K_d(\mathbf{r})\big[\rho_w\, g h(\mathbf{r}) \sin\beta(\mathbf{r}) - \tau_{cr}(\mathbf{r})\big]^q \qquad (13)$$

where $\tau(\mathbf{r}) = \rho_w\, g h(\mathbf{r}) \sin\beta(\mathbf{r})$ $[Pa]$ is the shear stress, β $[deg]$ is the slope angle, p and

q are exponents, $K_t(\mathbf{r})$ $[s]$ is the effective transport capacity coefficient, $K_d(\mathbf{r})$ $[s/m]$ is the effective erodibility (detachment capacity coefficient), $\rho_w\, g$ is the hydrostatic pressure of water with the unit height, $g = 9.81$ $[m/s^2]$ is the gravitational acceleration, $\rho_w = 10^3$ $[kg/m^3]$ is the mass density of water, and $\tau_{cr}(\mathbf{r})$ $[Pa]$ is the critical shear stress (in this project we assume $\tau_{cr} \approx 0$).

The parameters and adjustment factors for the estimation of $D_c(\mathbf{r}), T(\mathbf{r}), \sigma(\mathbf{r})$ are functions of soil and cover properties, and their values for a wide range of soils, cover, agricultural and erosion prevention practices are being developed within the WEPP model [Flanagan and Nearing, 1995]. WEPP manual describes the procedures used for the development of these parameters, and gives empirical equations representing relations between erodibility and soil texture and organic matter content. It also presents numerous adjustment factors reflecting the impact of vegetation and various erosion prevention measures and agricultural practices. WEPP program includes a data base which automatically assigns the values of parameters for the given soil, vegetation characteristics and practices. Similar database can be created for military land use, and thus simplify the task of selecting the proper parameters.

Our simulations have pointed out to some unsolved issues related to the interpretation and determination of the soil and cover parameters. Our effort to predict the observed extent of deposition indicates that the equation for estimating the sediment transport capacity may not be applicable to a wide range of situations and a more general equation suitable for overland flow over complex terrain is still needed, as suggested by several papers [e.g., Govers, 1991; Guy et al., 1991]. It is also important to note that the parameters n, K_t and K_d lump the influence of numerous physical properties of soil and cover which are not mutually independent, however, their functional relationship is not known. The ongoing experimental research [e.g., Flanagan and Nearing, 1995], as well as the presented simulations, can provide a better insight into the physical meaning of these parameters and improve the quantitative accuracy of the predictions.

To prepare the input representing soil and cover properties for erosion simulation it is necessary to provide the data in the form of a raster map with the resolution identical to the resolution of DEM. This is straightforward, if the data are in a raster or vector format, as usually simple GIS transformation programs for resampling and vector-to-raster transformation can be used. However, if the data are given in the form of measurements at irregularly distributed points, spatial interpolation should be used. While there exist a large number of methods for spatial interpolation, the spline and kriging methods offer the most flexibility and the highest accuracy. We have already described the regularized spline with tension (RST) in its multivariate form in our previous report (Mitasova et al. 1996b, PART I), therefore we just briefly describe the principles of kriging and compare the two approaches, from the point of view of practical applications.

3.2 Geostatistical approach to interpolation

The principles of geostatistics and interpolation by kriging are described in a large volume of literature [e.g., Matheron 1971, Journel and Huijbregts 1978, Burrough 1986, Isaaks and Srivastava 1989, Oliver and Webster 1990, Deutsch and Journel 1992, Cressie 1993], therefore only the basic notions are outlined.

Kriging is based on a concept of random functions: the surface or volume is assumed to be one realization of a random function with a certain spatial covariance [Matheron 1971, Journel and Huijbregts 1978]. Using the given data $z(\mathbf{r}_i)$ and an assumption of stationarity one can estimate a semivariogram $\gamma(\mathbf{h})$, defined as

$$\gamma(\mathbf{h}) = \frac{1}{2} Var\left[\{z(\mathbf{r}+\mathbf{h}) - z(\mathbf{r})\}\right] \approx \frac{1}{2N_h}\sum_{(ij)}^{N_h}[z(\mathbf{r}_i) - z(\mathbf{r}_j)]^2 \tag{3}$$

which is related to the spatial covariance $C(\mathbf{h})$ as

$$\gamma(\mathbf{h}) = C(0) - C(\mathbf{h}) \tag{4}$$

where $C(0)$ is the semivariogram value at infinity (sill). The summation in Eq. (3) runs over the number N_h of pairs of points which are separated by the vector $\mathbf{h}$ within a small tolerance $\Delta\mathbf{h}$ (size of histogram bin). For isotropic data, the semivariogram can be simplified into a radial function dependent on $|\mathbf{h}|$. The kriging literature provides a choice of functions which can be used as theoretical semivariograms (spherical, exponential, Gaussian, Bessel, etc.) [Cressie 1993]. The parameters of these functions are then optimized for the best fit to the experimental semivariogram.

The interpolated surface is then constructed using statistical conditions of unbiasedness and minimum variance. In its dual form [Matheron 1971, Hutchinson and Gessler 1993] the universal kriging interpolation function can be written as

$$F(\mathbf{r}) = T(\mathbf{r}) + \sum_{j=1}^{N} \lambda_j C(\mathbf{r} - \mathbf{r}_j) \tag{5}$$

where $T(\mathbf{r})$ represents its non-random component (drift) expressed as a linear combination of low-order monomials. The monomial and $\{\lambda_j\}$ coefficients are found by solving a system of linear equations [Hutchinson and Gessler 1993].

In general, kriging predicts values at points and blocks in d-dimensional space and enables incorporation of anisotropy. Various extensions enhance its flexibility and range of applicability [Cressie 1993, Deutsch and Journel 1992]. Co-kriging includes information about correlations of two or more attributes to improve the quality of interpolation while disjunctive kriging is used for applications where the probability that the measured values exceed certain treshold is of interest. For cases in which assumption of stationarity is deemed not to be valid, zonal kriging can be used [Burrough 1986]. Approaches for spatio-temporal kriging reflect the different behavior of the modelled phenomenon in the time dimension. Time is treated either as an additional dimension with geometric or zonal anisotropy, or as a combination of the space and time correlation functions with a space-time stationarity hypothesis.

Recent applications of geostatistics have de-emphasized the use of kriging as an interpolation and mapping tool while shifting the focus towards models of uncertainty that depend on the data values in addition to the data configuration [Deutsch and Journel

1992, Journel 1996]. A stochastic technique of conditional simulation is used to generate alternative, equally probable realizations of a surface, reproducing both data and the estimated covariance. From such a set of statistical samples one can estimate spatially dependent picture of uncertainty which is inherently in the data.

The main strength of kriging is in the statistical quality of predictions (e.g., unbiasedness) and ability to predict the spatial distribution of uncertainty. It is often used in mining and petroleum industry, geochemistry, geology, soil science, and ecology where its statistical properties are of a great value [Cressie 1993, Burrough 1991, Oliver and Webster 1990, Isaaks and Srivastava 1989]. It has been less successful for applications where local geometry and smoothness are the key issues and other methods proved to be competitive or even better [Hardy 1990, Deutsch and Journel 1992].

As it has been pointed out by several authors [Matheron 1981, Wahba 1990, Hutchinson and Gessler 1993, Cressie 1993] splines are formally equivalent to universal kriging with the choice of the covariance function determined by the seminorm $I(F)$. Therefore, many of the geostatistical concepts can be exploited within the spline framework. However, the physical interpretation of splines make their application easier and more intuitive.

3.3 Relation and differences between geostatistical and variational approach

Theoretical and practical issues of relationship between kriging and splines have been discussed in several papers [e.g., Matheron 1981, Wahba 1990, Cressie 1993, Hutchinson and Gessler 1993], therefore we present only a brief comment.

Kriging assumes that the spatial distribution of a geographic phenomenon can be modelled by a realization of a random function and uses statistical techniques to analyze the data (drift, covariance) and statistical criteria (unbiasedness and minimum variance) for predictions. However, **subjective** decisions are necessary (Journel 1996) such as judgement about stationarity, choice of function for theoretical variogram, etc.

In addition, often the data simply lack the information about important features of the modelled phenomenon, such as surface analytical properties or physically acceptable local geometries. As we have mentioned, kriging is the most successful for phenomena with a very strong random component and/or for estimation of statistical characteristics (uncertainty).

Splines rely on a physical model with flexibility provided by change of elastic properties of the interpolation function. Often, physical phenomena result from processes which minimize energy, with a typical example of terrain with its balance between gravitation force, soil cohesion and impact of climate. For these cases, splines proved to be rather successful. Moreover, splines provide enough flexibility for local geometry analysis which is often used as input to various process-based models.

However, most of the surfaces or volumes are neither stochastic, nor elastic medium, but they are results of a host of natural (fluxes, diffusion, ...) and/or socioeconomic processes. Therefore, each of the mentioned methods has a limited realm of applicability and, depending on the knowledge and experience of the user, proper choice of the method and its parameters can significantly affect the final results.

The use of kriging and splines is illustrated by the following example. To model the spatial distribution of sand content in soil, kriging and RST were used to approximate a 2m resolution raster from point data sampled at approximately $50\,m$ grid. Kriging was used with spherical (Figure 10a) and Gaussian (Figure 10b) variograms resulting in two quite different models of spatial distribution, while the differences in statistical fit were marginal. The gaussian covariance leads to extremely smooth surface while the spherical one produces cusps around the data points. For comparison we have interpolated the data using the RST, resulting in a surface with more complex structure, that the results of kriging (Figure 10c). This example clearly illustrates the need for additional information and/or expertise for selection of the most realistic model.

4. GIS support for volume modeling of soil data

To support modeling of infiltration processes and long term effects of erosion, a 3D model of soil properties is needed. This need is being addressed by expanding the capabilities of GIS for handling multidimensional data.

Implementation and initial testing of a 3D grid data file format for managing volumetric spatial data was completed. The storage format and programmer's interface routines we developed allows random access to compressed floating point double precision 3D data with caching. It is fully integrated within the GIS, using the established database hierarchy for header and data files. See the html documentation for the specification used. The resulting library was used to write several utility applications such as **r3.in.ascii, r3.out.ascii, r3.in.grid3, r3.mask, r3.null, r3.info, g3.region**. In addition, a program **r3.mkdspf**, reads the 3D grid data and creates a "display" file containing geometry for drawing isosurfaces to represent the data for visualization purposes. Future work will incorporate the 3D grid file and display file formats for use within visualization tools. Online documentation for this capabilities is available at

http://www.cecer.army.mil/grass/viz/htdoc/g3d/specification.html

and

http://www.cecer.army.mil/grass/viz/htdoc/g3d/protos.html.

Full report on visualization environment specially designed for development and communication of landscape process simulations based on multivariate fields will be provided as report for the contract DACA88-97-Q-0019. This environment is fully integrated with a GIS, and it is based on the OPEN GL standard allowing its portability. Its versions have been used throughout the projects described in reports (Mitasova et al. 1995b, 1996b, this report).

5. Conclusions

We have presented a new, distributed, process based erosion model based on the solution of continuity equation and implemented under the name SIMWE. This new model, developed by Mitas et al. [1996], Mitas et al. [1997], Mitas and Mitasova [submitted], simulates erosion/deposition patterns which represent transition from the detachment limited case (for small values of σ) towards the transport capacity limited case (large values of σ). This model therefore incorporates the type of erosion modeled by both the USLE and by the Unit Stream Power based model as its special cases and is consistent with the erosion model used by the WEPP.

While several important phenomena were not studied in this work and are the subject of our current research, the presented model and comparisons with the observed data have demonstrated the important role of terrain in the development of erosion/deposition patterns in the landscape. We have also shown that spatially variable cover can significantly change the general pattern of erosion/deposition and that the capability to simulate the cover's impact can become a powerful tool for a computer aided design of cost effective erosion protection measures.

The proposed erosion simulations tools are supported by numerous GIS functions, such as interpolation and visualization, and by new data structures which had been significantly enhanced within this project.

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